

Relativistic correction to $e^+e^- \rightarrow J/\psi + gg$ at B factories and constraint on color-octet matrix elements

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Abstract

We calculate the relativistic correction to J/ψ production in the color-singlet process $e^+e^- \rightarrow J/\psi + gg$ at B -factories. We employ the nonrelativistic QCD factorization approach, where the short-distance coefficients are calculated perturbatively and the long-distance matrix elements are extracted from the decays of J/ψ into e^+e^- and light hadrons. We find that the $O(v^2)$ relativistic correction can enhance the cross section by a factor of 20-30%, comparable to the enhancement due to the $O(\alpha_s)$ radiative correction obtained earlier. Combining the relativistic correction with the QCD radiative correction, we find that the color-singlet contribution to $e^+e^- \rightarrow J/\psi + gg$ can saturate the latest observed cross section $\sigma(e^+e^- \rightarrow J/\psi + X_{\text{non-}c\bar{c}}) = 0.43 \pm 0.09 \pm 0.09$ pb by Belle, thus leaving little room to the color-octet contributions. This gives a very stringent constraint on the color-octet contribution, and may imply that the values of color-octet matrix elements are much smaller than expected earlier by using the naive velocity scaling rules or extracted from fitting experimental data with the leading-order calculations.

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I. INTRODUCTION

The nonrelativistic QCD (NRQCD) effective field theory, introduced in 1995 by Bodwin, Braaten, and Lepage[1], has been widely accepted as a rigorous approach to study the production and decay of heavy quarkonium, the bound state of heavy quark Q and antiquark \bar{Q} pair (see Ref.[2] for a review). In the framework of NRQCD the production of heavy quarkonium is factorized into two parts, the short-distance part and the long-distance part. In the short-distance part, the $Q\bar{Q}$ pair is created in certain J^{PC} and color states, which can be calculated perturbatively through the expansion of QCD coupling constant α_s . The long-distance part describes the evolution of the $Q\bar{Q}$ pair into the physical hadron states through the emission of soft gluons with the corresponding universal nonperturbative matrix elements, which are weighted by powers of the relative velocity v_Q of heavy quarks in the meson rest frame. One intrinsic character of NRQCD is the inclusion of the effect of $Q\bar{Q}$ pair in a color-octet state, *i.e.* the color-octet mechanism. Since in the e^+e^- collision, the structure of the parton involved is simpler and the signals can be prominent, it is a good place to study the heavy quarkonium production and test the color-octet mechanism.

In recent years, the cross sections of inclusive J/ψ production in e^+e^- annihilation at $\sqrt{s} = 10.6\text{GeV}$ have been reported by BaBar[3], Belle[4], and CLEO[5] collaborations with the results

$$\sigma(e^+e^- \rightarrow J/\psi + X) = 2.5 \pm 0.21 \pm 0.21\text{pb}, \quad (1a)$$

$$\sigma(e^+e^- \rightarrow J/\psi + X) = 1.47 \pm 0.10 \pm 0.13\text{pb}, \quad (1b)$$

$$\sigma(e^+e^- \rightarrow J/\psi + X) = 1.9 \pm 0.2\text{pb}, \quad (1c)$$

respectively. If the J/ψ 's momentum p_ψ^* is restricted to $p_\psi^* > 2\text{GeV}/c$, the results of BaBar[3] and Belle[4] become

$$\sigma(e^+e^- \rightarrow J/\psi + X) \Big|_{p_\psi^* > 2\text{GeV}/c} = 1.87 \pm 0.10 \pm 0.15\text{pb}, \quad (2a)$$

$$\sigma(e^+e^- \rightarrow J/\psi + X) \Big|_{p_\psi^* > 2\text{GeV}/c} = 1.05 \pm 0.04 \pm 0.09\text{pb}. \quad (2b)$$

It can be seen that these experimental measurements are not entirely consistent with each other. The two B -factories also measured the J/ψ momentum distributions and observed

similar shapes. Despite of the disagreement among different experimental measurements, theoretically the inclusive J/ψ production has been extensively investigated within the color-singlet model[6–9] and the color-octet mechanism in NRQCD[9–11]. In NRQCD, one important contribution to the inclusive J/ψ production in e^+e^- annihilation at $\sqrt{s} = 10.6$ GeV comes from the color-octet process $e^+e^- \rightarrow J/\psi[{}^1S_0^{(8)}, {}^3P_J^{(8)}] + g$ [9, 10]. But its predictions of the J/ψ enhancement near the kinematics end point region is not observed. After applying the soft-collinear effective theory (SCET)[12], the shape of J/ψ momentum distribution can be softened. However this depends phenomenologically on a nonperturbative shape function.

Experimentally, the $e^+e^- \rightarrow J/\psi + X$ process can be divided into two parts: the $e^+e^- \rightarrow J/\psi + c\bar{c} + X$ part and the $e^+e^- \rightarrow J/\psi + X_{\text{non-}c\bar{c}}$ part. The Belle collaboration finds the ratio[13]

$$R_{c\bar{c}} = \frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \rightarrow J/\psi + X)} = 0.59_{-0.13}^{+0.15} \pm 0.12. \quad (3)$$

which corresponds to $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.87_{-0.19}^{+0.21} \pm 0.17$ pb. The latest measurement of J/ψ production in association with a $c\bar{c}$ pair carried out by Belle gives[14]

$$\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.74 \pm 0.08_{-0.08}^{+0.09} \text{ pb}. \quad (4)$$

The experimental results are more than 5 times larger than leading-order (LO) NRQCD predictions[7, 8, 15]. This large discrepancy could be resolved by including the NLO QCD corrections and the feed-down of higher excited states[16, 17]. Belle also analyzed the $e^+e^- \rightarrow J/\psi + X_{\text{non-}c\bar{c}}$ process and obtained[14]

$$\sigma(e^+e^- \rightarrow J/\psi + X_{\text{non-}c\bar{c}}) = 0.43 \pm 0.09 \pm 0.09 \text{ pb}. \quad (5)$$

At LO in α_s the non- $c\bar{c}$ process via color-singlet channels only includes $e^+e^- \rightarrow J/\psi + gg$. The theoretical prediction at LO in α_s and v^2 is about 0.2pb[9], and recent works[18, 19] find that the K-factor of the NLO correction is about 1.2-1.3. In charmonium system the relative velocity v_c of a $c\bar{c}$ pair in the J/ψ rest frame is not small. The value of v_c^2 is about 0.3, which is close to the size of $\alpha_s(m_c)$. So the relativistic effect may also be important. Our previous work[20] shows that in the $e^+e^- \rightarrow J/\psi + \eta_c$ exclusive process, the relativistic correction is indeed important in resolving the more than one order of magnitude discrepancies between experimental data[21, 22] and LO NRQCD predictions[23]. We find when combining the relativistic correction together with the NLO QCD corrections[24] the conflict between experimental measurement and theoretical prediction is almost resolved (see

also Ref. [25] for a similar result). However, in the $e^+e^- \rightarrow J/\psi + c\bar{c}$ process, we find the relativistic correction is very small and may be ignored. And the recent work shows[26] that in the $p\bar{p} \rightarrow J/\psi + X$ process at the Tevatron the relativistic corrections can also be neglected. Then it is necessary to investigate the relativistic correction to the $e^+e^- \rightarrow J/\psi + gg$ process and clarify how large the relativistic correction is and whether it is positive or negative. In this work we will deal with this problem within the framework of NRQCD factorization approach, and in particular, we will examine the effect of the relativistic correction on the constraint on color-octet matrix elements. The rest of this paper is organized as follows. In Sec.II, we will introduce the NRQCD factorization formula and describe how we calculate the short-distance coefficients and determine the long-distance matrix elements. We will present our calculations and show our result in Sec.III. Discussions and a summary will be given in the last section.

II. THE NRQCD FACTORIZATION FORMULA

According to NRQCD[1] effective theory, up to $\mathcal{O}(v^2)$ the inclusive J/ψ production rate can be expressed as

$$\sigma(e^+e^- \rightarrow J/\psi + X) = \frac{F_1(^3S_1)}{m_c^2} \langle 0 | \mathcal{O}_1^\psi(^3S_1) | 0 \rangle + \frac{G_1(^3S_1)}{m_c^4} \langle 0 | \mathcal{P}_1^\psi(^3S_1) | 0 \rangle + \mathcal{O}(v^4\sigma). \quad (6)$$

where

$$\mathcal{O}_1^\psi(^3S_1) = \chi^\dagger \sigma^i \psi (a_\psi^\dagger a_\psi) \psi^\dagger \sigma^i \chi \quad (7)$$

and

$$\mathcal{P}_1^\psi(^3S_1) = \frac{1}{2} [\chi^\dagger \sigma^i \psi (a_\psi^\dagger a_\psi) \psi^\dagger \sigma^i (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \chi + \chi^\dagger \sigma^i (-\frac{i}{2} \overleftrightarrow{\mathbf{D}})^2 \psi (a_\psi^\dagger a_\psi) \psi^\dagger \sigma^i \chi] \quad (8)$$

are the four-fermion operators with dimensions six and eight, respectively. $F_1(^3S_1)$ and $G_1(^3S_1)$ are the corresponding short-distance coefficients. The short-distance coefficients can be obtained perturbatively through the matching condition

$$\sigma(Q\bar{Q}) \Big|_{\text{pert QCD}} = \sum_n \frac{F_n(\Lambda)}{m_c^{d_n-4}} \langle 0 | \mathcal{O}_n^{Q\bar{Q}}(\Lambda) | 0 \rangle \Big|_{\text{pert NRQCD}} \quad (9)$$

The long-distance matrix elements characterized by the velocity v_c can be estimated by lattice calculations or phenomenological models, or determined by fitting experimental data.

In this work, the covariant spinor projection method[27] is adopted to evaluate the left-hand side of Eq. [9]. In this method, the Dirac spinor product $v(P/2 - q)\bar{u}(P/2 + q)$ is

projected onto a certain $^{(2S+1)}L_J$ state in a Lorentz covariant form (see, e.g., [28, 29]), which makes the short-distance coefficients evaluated directly. In the J/ψ (with $S = 1$) case, the expression of the spinor production projection in the meson rest frame up to all orders of v_c^2 is [29]

$$\sum_{\lambda_1 \lambda_2} v(-\mathbf{q}, \lambda_2) \bar{u}(\mathbf{q}, \lambda_1) \langle \frac{1}{2}, \lambda_1; \frac{1}{2}, \lambda_2 | 1, \epsilon \rangle = \frac{1}{\sqrt{2}} (E + m) (1 - \frac{\boldsymbol{\alpha} \cdot \mathbf{q}}{E + m}) \boldsymbol{\alpha} \cdot \boldsymbol{\epsilon} \frac{1 + \gamma_0}{2} (1 + \frac{\boldsymbol{\alpha} \cdot \mathbf{q}}{E + m}) \gamma_0. \quad (10)$$

And in an arbitrary frame, it becomes

$$\sum_{\lambda_1 \lambda_2} v(q, \lambda_2) \bar{u}(q, \lambda_1) \langle \frac{1}{2}, \lambda_1; \frac{1}{2}, \lambda_2 | 1, \epsilon \rangle = -\frac{1}{2\sqrt{2}(E + m)} (\frac{1}{2} \not{P} - \not{q} - m) \not{\epsilon} \frac{\not{P} + 2E}{2E} (\frac{1}{2} \not{P} + \not{q} + m). \quad (11)$$

Here the normalization of the Dirac spinors is $\bar{u}u = -\bar{v}v = 2m_c$, and the relations between momenta of quark and antiquark in an arbitrary frame and in the meson rest frame are given by [30]

$$\frac{1}{2}P + q = L(\frac{1}{2}P_r + \mathbf{q}), \quad (12a)$$

$$\frac{1}{2}P - q = L(\frac{1}{2}P_r - \mathbf{q}), \quad (12b)$$

where $P_r^\mu = (2E_q, \mathbf{0})$, $E_q = \sqrt{m^2 + \mathbf{q}^2}$, and $2\mathbf{q}$ is the relative momentum between two quarks in the meson rest frame. L_μ^v is the boost tensor from the meson rest frame to an arbitrary frame.

Then the cross section for $e^+e^- \rightarrow J/\psi + gg$ up to next-to-leading order in v^2 can be expressed as

$$\sigma(e^+e^- \rightarrow J/\psi + gg) = \frac{\langle 0 | \mathcal{O}_1^\psi(^3S_1) | 0 \rangle}{3} \frac{1}{2s} \int \bar{N}_0 d\Phi_3 + \frac{\langle 0 | \mathcal{P}_1^\psi(^3S_1) | 0 \rangle}{3} \frac{1}{2s} \int \bar{N}_1 d\Phi_3, \quad (13)$$

where Φ_3 is the three-body phase space, bar means averaging the spins over the initial states and summing up the spins over the final states. The short-distance part \bar{N}_0 and \bar{N}_1 defined in Eq.[17,18] can be calculated perturbatively. The numerical values of the two long-distance matrix elements $\langle 0 | \mathcal{O}_1^\psi(^3S_1) | 0 \rangle$ and $\langle 0 | \mathcal{P}_1^\psi(^3S_1) | 0 \rangle$ will be estimated by nonperturbative methods. In the nonrelativistic limit, $\langle 0 | \mathcal{O}_1^\psi(^3S_1) | 0 \rangle$ can be related to the nonrelativistic bound state wave function at the origin. And in NRQCD effective theory,

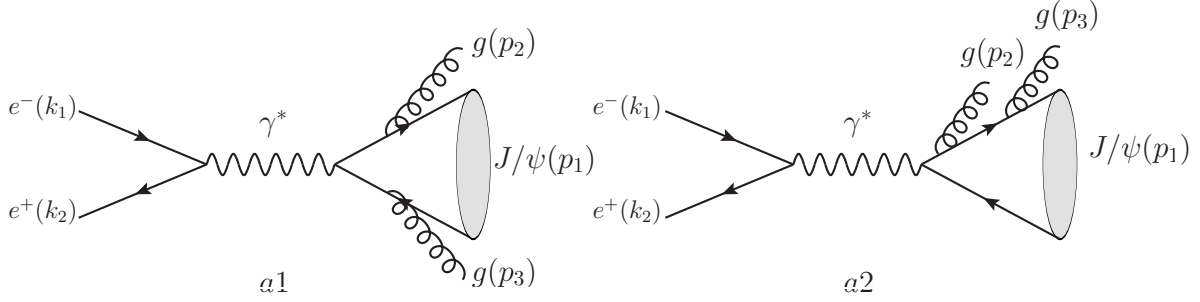


FIG. 1: The typical two among six Feynman diagrams in the process $e^+e^- \rightarrow J/\psi + gg$ at leading-order in QCD.

$\langle 0 | \mathcal{P}_1^\psi(^3S_1) | 0 \rangle$ also can be calculated with the help of potential model[31]. Here we determine them with an alternate way by extracting them from the decays of J/ψ into light hadrons and into e^+e^- .

III. RELATIVISTIC CORRECTIONS TO $e^+e^- \rightarrow J/\psi + gg$

A. Short-Distance Coefficients

At LO in QCD, there are six Feynman diagrams in the process $e^+e^- \rightarrow J/\psi + gg$, and the typical two are shown in Fig.[1]. The full QCD amplitude for this process can be expanded in terms of the quark relative momentum q_ψ^α :

$$M(e^+e^- \rightarrow (C\bar{C})_{3S_1}(P_{J/\psi}) + gg) = \left(\frac{m_c}{E}\right)^{1/2} A(q_\psi) = \left(\frac{m_c}{E}\right)^{1/2} \left(A(0) + q_\psi^\alpha \frac{\partial A}{\partial q_\psi^\alpha} \Big|_{q_\psi=0} + \frac{1}{2} q_\psi^\alpha q_\psi^\beta \frac{\partial^2 A}{\partial q_\psi^\alpha \partial q_\psi^\beta} \Big|_{q_\psi=0} + \dots \right), \quad (14)$$

where

$$A(q_\psi) = \sum_{\lambda_1 \lambda_2} \sum_{ij} \left\langle \frac{1}{2}, \lambda_1, \frac{1}{2}, \lambda_2 | 1, S_z \right\rangle \langle 3, i; \bar{3}, j | 1 \rangle A(e^+e^- \rightarrow c_{\lambda_1, i} \left(\frac{P}{2} + q_\psi\right) \bar{c}_{\lambda_2, j} \left(\frac{P}{2} - q_\psi\right) + gg), \quad (15)$$

where $\langle 3, i; \bar{3}, j | 1 \rangle = \delta_{ij} / \sqrt{N_c}$ is the color-SU(3) Clebsch-Gordon coefficient for a $c\bar{c}$ pair projecting onto a color-singlet state. With the help of Eq.[11], we can express $A(q_\psi)$ in a covariant form. The factor $\left(\frac{m_c}{E}\right)^{1/2}$ comes from the relativistic normalization of the $c\bar{c}$ state, and $E = \sqrt{m_c^2 + \mathbf{q}_\psi^2}$.

For the S wave state the odd-power terms of $q_{J/\psi}$ vanish and $q^\alpha q^\beta = \frac{\mathbf{q}^2}{3}(-g^{\alpha\beta} + \frac{P^\alpha P^\beta}{P^2}) = \frac{\mathbf{q}^2}{3}\Pi^{\alpha\beta}$, where $P^2 = 4E^2$, $P \cdot q = 0$. Then at LO of v^2 we have^a

$$|M|^2 = \frac{m_c}{E_1} A(0) A^*(0) + \frac{1}{2} q_\psi^\alpha q_\psi^\beta A_{\alpha\beta} A^*(0) + \frac{1}{2} q_\psi^\alpha q_\psi^\beta A_{\alpha\beta}^* A(0), \quad (16)$$

where $A_{\alpha\beta} = \frac{\partial^2 A}{\partial q^\alpha \partial q^\beta}$, and $A_{\alpha\beta}^* = \frac{\partial^2 A^*}{\partial q^\alpha \partial q^\beta}$. According to the spinor projection method the short-distance part of the v_c^0 part is

$$\overline{N}_0 = \frac{1}{2N_c m_c} (A(0) A^*(0)) \Big|_{\mathbf{q}_{J/\psi}^2=0}, \quad (17)$$

and the corresponding v_c^2 part is

$$\overline{N}_1 = \frac{1}{2N_c m_c} \left(\frac{\partial(\frac{m_c}{E} A(0) A^*(0))}{\partial(\mathbf{q}_\psi^2)} \Big|_{\mathbf{q}_{J/\psi}^2=0} + \frac{1}{6} \Pi_{\alpha\beta} (A_\psi^{\alpha\beta} A^*(0) + A(0) A_\psi^{*\alpha\beta}) \Big|_{\mathbf{q}_{J/\psi}^2=0} \right). \quad (18)$$

We introduce the dimensionless variables $z_i = 2E_i/\sqrt{s}$, $\vec{q}_i = 2\vec{p}_i/\sqrt{s}$, $x_i = \cos\theta_i$ and $\delta = 4m_c/\sqrt{s}$ to describe the $e^-(k_1) + e^+(k_2) \rightarrow J/\psi(p_1) + g(p_2) + g(p_3)$ process. Here \sqrt{s} is the total energy in the center of mass frame, p_1^μ , p_2^μ , p_3^μ are the four-momenta of the final state J/ψ and the two gluons respectively, and θ_i is the angle between state i and the electron. The scalar products between the momenta can be expressed as

$$\begin{aligned} k_1 \cdot p_1 &= \frac{s}{4}(z_1 - q_1 x_1); \quad k_2 \cdot p_1 = \frac{s}{4}(z_1 + q_1 x_1); \quad p_2 \cdot p_3 = \frac{s}{8}(4 - 4z_1 + \delta^2); \\ k_1 \cdot p_2 &= \frac{s}{4}(z_2 - (q_- x_- - q_1 x_1)/2); \quad k_2 \cdot p_2 = \frac{s}{4}(z_2 + (q_- x_- - q_1 x_1)/2); \\ k_1 \cdot p_3 &= \frac{s}{4}(z_3 + (q_- x_- + q_1 x_1)/2); \quad k_2 \cdot p_3 = \frac{s}{4}(z_3 - (q_- x_- + q_1 x_1)/2); \\ p_1 \cdot p_2 &= \frac{s}{8}(4 - \delta^2 - 4z_3); \quad p_1 \cdot p_3 = \frac{s}{8}(4 - \delta^2 - 4z_2); \quad k_1 \cdot k_2 = \frac{s}{2}; \end{aligned} \quad (19)$$

where $z_- = z_2 - z_3$, $q_- = |\vec{q}_2 - \vec{q}_3| = \sqrt{4 - 4z_1 + \delta^2 + z_-^2}$, $q_1 = |\vec{q}_1| = \sqrt{z_1 - \delta^2}$, $x_- = \cos\theta_-$, and θ_- is the angle between \vec{q}_- and the electron. And the three-body phase space is then given by

$$\begin{aligned} d\Phi_3 &= (2\pi)^4 \delta^4(k_1 + k_2 - p_1 - p_2 - p_3) \prod_{i=1}^3 \frac{d^3 p_i}{2E_i} \\ &= \frac{s}{32(2\pi)^4} \frac{dz_1 dx_1 dz_- dw}{\sqrt{(1 - K^2)(1 - x_1^2) - w^2}}, \end{aligned} \quad (20)$$

^a We do not expand the three-body phase space by v_c^2 and assume $M_{J/\psi} = 2m_c$.

where

$$K = \frac{z_-(2 - z_1)}{q_1 q_-}, \quad (21a)$$

$$w = x_- + K x_1. \quad (21b)$$

The ranges of those integral variables are

$$\delta \leq z_1 \leq 1 + \frac{\delta^2}{4}, \quad (22a)$$

$$-1 \leq x_1 \leq 1, \quad (22b)$$

$$-\sqrt{(z_1^2 - \delta^2)} \leq z_- \leq \sqrt{(z_1^2 - \delta^2)}, \quad (22c)$$

$$-\sqrt{(1 - K^2)(1 - x_1^2)} \leq w \leq \sqrt{(1 - K^2)(1 - x_1^2)}. \quad (22d)$$

B. Long-Distance Matrix Elements

The color-singlet production matrix elements can be related to the decay matrix elements in the vacuum saturation approximation[1] and the errors are of v_c^4 order. Since in this $e^+e^- \rightarrow J/\psi + gg$ process, there are two NRQCD matrix elements accurate to order v_c^2 , i.e., $\langle 0 | \mathcal{O}_1^\psi(^3S_1) | 0 \rangle$ and $\langle 0 | \mathcal{P}_1^\psi(^3S_1) | 0 \rangle$, we can determine their values by fitting J/ψ decays into e^+e^- and into light hadrons (LH). The theoretical results at NLO in α_s and v^2 for $J/\psi \rightarrow e^+e^-$ and $J/\psi \rightarrow LH$ ^b are [29]

$$\Gamma(J/\psi \rightarrow e^+e^-) = \frac{2e_c^2 \pi \alpha^2}{3} \left(\left(1 - \frac{16\alpha_s}{3\pi}\right) \frac{\langle 0 | \mathcal{O}_1^\psi(^3S_1) | 0 \rangle / 3}{m_c^2} - \frac{4}{3} \frac{\langle 0 | \mathcal{P}_1^\psi(^3S_1) | 0 \rangle / 3}{m_c^4} \right), \quad (23a)$$

$$\Gamma(J/\psi \rightarrow LH) = \left(\frac{20\alpha_s^3}{243} (\pi^2 - 9) \right) \left(\left(1 - 2.55 \frac{\alpha_s}{\pi}\right) \frac{\langle 0 | \mathcal{O}_1^\psi(^3S_1) | 0 \rangle / 3}{m_c^2} - \frac{19\pi^2 - 132}{12\pi^2 - 108} \frac{\langle 0 | \mathcal{P}_1^\psi(^3S_1) | 0 \rangle / 3}{m_c^4} \right). \quad (23b)$$

And the central values of the experimental results are[32]

$$\Gamma(J/\psi \rightarrow e^+e^-) = 5.54 \text{keV}, \quad \Gamma(J/\psi \rightarrow LH) = 69.2 \text{keV} \quad (24)$$

Solving these equations at LO of α_s (QCD radiative corrections not included), we obtain

$$\frac{\langle 0 | \mathcal{O}_1^\psi(^3S_1) | 0 \rangle}{3} = 0.294 \text{GeV}^3, \quad \frac{\langle 0 | \mathcal{P}_1^\psi(^3S_1) | 0 \rangle}{3m_c^2} = 0.320 \times 10^{-1} \text{GeV}^3, \quad (25)$$

^b We do not include the electromagnetic process $J/\psi \rightarrow \gamma^* \rightarrow LH$.

for $m_c = 1.5\text{GeV}$ and $\alpha_s = 0.26$. Fixing $\alpha_s = 0.26$, we can express the matrix elements as functions of m_c , which are

$$\frac{\langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle}{3} = 0.131m_c^2, \quad \frac{\langle 0|\mathcal{P}_1^\psi(^3S_1)|0\rangle}{3m_c^2} = 0.142 \times 10^{-1}m_c^2. \quad (26)$$

If we include the QCD NLO radiative corrections in Eq.[23], for $m_c = 1.5\text{GeV}$ and $\alpha_s = 0.26$, we get

$$\frac{\langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle}{3} = 0.572\text{GeV}^3, \quad \frac{\langle 0|\mathcal{P}_1^\psi(^3S_1)|0\rangle}{3m_c^2} = 0.512 \times 10^{-1}\text{GeV}^3. \quad (27)$$

The corresponding m_c dependence of the matrix elements are

$$\frac{\langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle}{3} = 0.254m_c^2, \quad \frac{\langle 0|\mathcal{P}_1^\psi(^3S_1)|0\rangle}{3m_c^2} = 0.228 \times 10^{-1}m_c^2. \quad (28)$$

In Ref.[33], the authors relate $\langle 0|\mathcal{P}_1^\psi(^3S_1)|0\rangle$ to $\langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle$ using the NRQCD equation of motion. In Ref.[31], $\langle 0|\mathcal{P}_1^\psi(^3S_1)|0\rangle$ is calculated by combining NRQCD and the potential model. Alternately, we estimate the production matrix elements by using the experimentally observed charmonium decay rates. As argued in[1], the differences between the color-singlet production and decay matrix elements are of order v^4 . So our method should be valid at order v^2 , and our numerical results of the production matrix elements are adequate, if the higher order QCD and v^2 corrections to the decay rates are small and the uncertainties of the experimental data are not large. If we define $O(\langle v^2 \rangle) = \frac{\langle 0|\mathcal{P}_1^\psi(^3S_1)|0\rangle}{m_c^2 \langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle}$, we find the value of $O(\langle v^2 \rangle)$ to be about 0.1 from Eq.(25) or Eq.(27), which is about 2 \sim 3 times smaller than the result calculated within a potential model[31, 34] or within the QCD sum rules[35]. Note that, differing from theirs, our matrix elements are extracted by fitting experimental data, which depend on the chosen processes and experimental errors. Moreover, the short-distance coefficient of the v^2 correction term in Eq.(23b) for $J/\psi \rightarrow \text{LH}$ is quite large, implying that the corresponding long-distance matrix element could be rather small. These of course will have uncertainties, compared with other methods for calculating the long-distance matrix elements. Nevertheless, our method using experimental data to extract the matrix elements, provides an independent and self-consistent way to determine the matrix elements. So in this work we will use these experimentally extracted long-distance matrix elements to give numerical predictions.

C. Numerical Result

The expression of N_0 and N_1 are too complicated to be shown here and we only give the analytical expression of the differential cross section. With J^{PC} conservation and gauge invariance, the general form of the differential cross section of unpolarized J/ψ production in e^+e^- annihilation can be expressed as

$$\frac{d^2\sigma}{dz_1 dx_1}(e^+e^- \rightarrow \gamma^* \rightarrow \psi + gg) = S_0(z_1)(1 + \alpha_0(z_1)x_1^2) + S_v(z_1)(1 + \alpha_v(z_1)x_1^2), \quad (29)$$

where the first term on the right-hand side of Eq.[29] is the LO result in v_c , and the second one is the relativistic correction term. Our results of $S_0(z_1)$ and $\alpha_0(z_1)$ are in consistency with those in [7]. And the expressions of $S_v(z_1)$ and $S_v(z_1)\alpha_v(z_1)$ are too complicated and they will not be given here.

Setting $\alpha_s = 0.26$, $\sqrt{s} = 10.6\text{GeV}$ and $m_c = 1.5\text{GeV}$ and integrating out x_1 and z_1 numerically, we find when choosing the values of the matrix elements in Eq.[25], the LO result is

$$\sigma_{LO}(e^+ + e^- \rightarrow J/\psi + gg) = 202 \text{ fb}. \quad (30)$$

and the relativistic correction is 55 fb, which gives about 27% enhancement, then resulting in the NLO result in v_c^2 :

$$\sigma_{\text{NLO}(v_c^2)}(e^+ + e^- \rightarrow J/\psi + gg) = 257 \text{ fb}. \quad (31)$$

If we choose the values of the matrix elements in Eq.[27], the NLO result in v_c^2 becomes

$$\sigma_{\text{NLO}(v_c^2)}(e^+ + e^- \rightarrow J/\psi + gg) = 480 \text{ fb}, \quad (32)$$

and the relativistic correction enhancement is 1.22. When the charm-quark mass varies from 1.4GeV to 1.6GeV, the LO and NLO cross sections as function of m_c are shown in Fig.[2] with the long-distance matrix elements, respectively, in Eq.(26) and Eq.(28). As mentioned above, we do not expand the three-body phase space by v_c^2 and assume $M_{J/\psi} = 2m_c$ for simplicity. Instead, if we used the physical mass of $M_{J/\psi} = 3.097\text{GeV}$, which includes v_c^2 kinematic and binding energy corrections, when doing the phase space integrals, the results given in Eq.(31) and Eq.(32) turned to be 253fb and 475fb, respectively. Comparing those results, one can see that the differences due to replacing $M_{J/\psi} = 3.097\text{GeV}$ with $M_{J/\psi} = 2m_c$ in the phase space integration can be neglected.

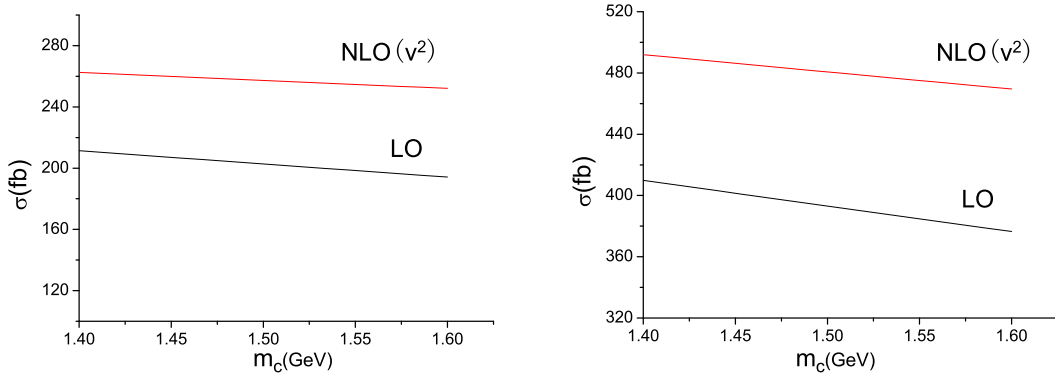


FIG. 2: Relativistic corrections to $\sigma(e^+e^- \rightarrow J/\psi + gg)$ as functions of m_c with long-distance matrix elements determined from Eq.(26) (left-panel) and Eq.(28) (right-panel). In each figure the lower curve is the LO result and the upper one is the result including v_c^2 corrections.

To be consistent, when we calculate the production cross section including both the $O(v^2)$ relativistic correction and the $O(\alpha_s)$ radiative correction, we should first use the relativistic correction results obtained by adopting Eqs.(27,28), where the $O(\alpha_s)$ radiative corrections to the matrix elements are included, and then further include the $O(\alpha_s)$ radiative corrections to the production short-distance coefficients. This will be discussed in next section.

IV. DISCUSSION AND SUMMARY

In above the order v_c^2 relativistic effect is considered for the color-singlet $e^+e^- \rightarrow J/\psi + gg$ process in the framework of NRQCD. We calculate the short-distance coefficients perturbatively and find the ratio of the coefficient of $\langle 0|\mathcal{P}_1^\psi(^3S_1)|0\rangle/m_c^2$ to that of $\langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle$ is about 2.5. Denoting the ratio of the long-distance matrix elements by $\langle 0|\mathcal{P}_1^\psi(^3S_1)|0\rangle/(m_c^2\langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle) = \langle v_c^2\rangle$ [31], then the enhancement of the relativistic correction can be expressed by $2.5\langle v_c^2\rangle$. Using the matrix elements given in Eq.[25] and Eq.[27] as inputs, we predict the enhancements of relativistic corrections are 22% and 27%, respectively, which are as important as the NLO QCD corrections[18]. It can also be found that when including the relativistic corrections the m_c dependence is also improved a little. If we determine the matrix element from the $J/\psi \rightarrow e^+e^-$ process in Eq.[23a] without including the relativistic and QCD corrections, we get $\frac{\langle 0|\mathcal{O}_1^\psi(^3S_1)|0\rangle}{3} = 0.251\text{GeV}^3$ and the LO result of

$e^+e^- \rightarrow J/\psi + gg$ is only about 173fb. Then we can see that the relativistic corrections can enhance both the short-distance coefficients and the long-distance matrix elements.

Next, we further include the $O(\alpha_s)$ radiative corrections to the production short-distance coefficients. In Ref.[18], the authors obtain that with $\alpha_s(2m_c) = 0.259$ the K-factor of NLO QCD result to LO QCD result is 1.20. Then using the matrix elements in Eq.[27], which also include the NLO QCD corrections in decay processes, we find that after including the QCD corrections[18] the combined cross section $\sigma_{\text{NLO}(v_c^2, \alpha_s)}$ for $m_c = 1.5\text{GeV}$ is

$$\sigma_{\text{NLO}(v_c^2, \alpha_s)}(e^+e^- \rightarrow J/\psi + gg) \simeq 480/1.22 \times (1 + 0.22 + 0.20) \simeq 560\text{fb}, \quad (33)$$

where on the right-hand side of Eq.(33) the number 480 fb comes from Eq.(32), and in the summation the enhancement factor 0.22 is due to relativistic correction, while the enhancement factor 0.20 due to QCD radiative correction. Note that all the above contributions come from the color-singlet part. The LO color-octet contribution of $e^+e^- \rightarrow J/\psi + g$ can be estimated as large as 0.27pb[11], but this apparently depends on the chosen values of the color-octet matrix elements. If using this estimate[11] for the color-octet contribution, then the prediction of NRQCD for the $J/\psi + X_{\text{non-c}\bar{c}}$ cross section at B -factories would become 0.83pb, which is almost twice as large as the measured central value 0.43pb by Belle[14].

In fact, from Eq.(33) we see that after including the QCD and relativistic corrections the color-singlet contribution alone has saturated the measured value of $J/\psi + X_{\text{non-c}\bar{c}}$ cross section, and thus there seems no need for the color-octet contribution. However, we must pay attention to possible uncertainties before we can draw a firm conclusion. First, on the experimental side, there is a large uncertainty of Belle's result in the $p_{J/\psi} < 2.0\text{GeV}$ region, and furthermore the total cross sections of inclusive J/ψ production measured by BaBar, Belle, and CLEO are not consistent. Second, on the theoretical side, we should take into account the uncertainty due to the choice of renormalization scale μ in the calculation of the short-distance coefficients. In doing the latter, we choose the largest value $\mu = \sqrt{s}/2$ and $m_c = 1.5\text{GeV}$, and find the K-factor of NLO QCD correction to be 1.48 (see Ref.[18]). We then combine the relativistic correction with the QCD radiative correction, and find that at NLO in v_c^2 and α_s the total cross section of direct J/ψ production becomes

$$\sigma_{\text{NLO}(v_c^2, \alpha_s)}(e^+e^- \rightarrow J/\psi + gg) \simeq 437\text{fb} \quad (34)$$

for $\mu = \sqrt{s}/2$ and $m_c = 1.5\text{GeV}$. We see that although the cross section is decreased as

compared with that for $\mu = 2m_c$ and $m_c = 1.5\text{GeV}$, the predicted cross section is still a little larger than the central value of the latest Belle result.

Moreover, the cross sections obtained in Eq.(33) and Eq.(34) are the direct J/ψ production rates, not including the feed-down contribution from higher charmonium states. If the feed-down contribution is included, the prompt J/ψ production cross section, which is the measured value by Belle, will be further enhanced by a factor of about 1.3 (see Refs.[18, 19] for discussions on the feed-down contribution). Therefore, the theoretical cross section for the J/ψ prompt production calculated at NLO in α_s and v^2 in NRQCD will exceed or saturate the latest value in Eq.(5) observed by Belle[14], despite of theoretical uncertainties related to the choice of input parameters, e.g., μ , m_c , and the color-singlet matrix elements.

In summary, we find the $O(v^2)$ relativistic correction to enhance the cross section of J/ψ production in the color-singlet process $e^+e^- \rightarrow J/\psi + gg$ by a factor of 20-30%, which is comparable to the enhancement caused by the $O(\alpha_s)$ radiative correction[18, 19]. As the consequence of including both the $O(\alpha_s)$ radiative correction and the $O(v^2)$ relativistic correction, the color-singlet contribution to $e^+e^- \rightarrow J/\psi + gg$ has saturated the latest observed cross section by Belle for $e^+e^- \rightarrow J/\psi + X_{\text{non-}c\bar{c}}$ at B -factories, thus leaving little room for the color-octet contribution. This gives a very stringent constraint on the color-octet contribution, and may imply that the values of color-octet matrix elements are much smaller than expected earlier by using the naive velocity scaling rules or extracted from fitting experimental data with the LO results. To reduce the theoretical uncertainties, further investigations for the higher order (both in α_s and v^2) corrections are needed. Moreover, comparisons between various experimental measurements and theoretical predictions are certainly helpful to clarify this important issue concerning the color-octet mechanism.

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